Dr. Anny Kumari

Dept. of physics

H. D. Jain College, Ara

PG I semester

Unit - 4

Topic - Group theory

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Group theory

Group :-

Let G be a non-empty set and $*$ be a binary operation defined on it, then the structure $(G, *)$ is said to be a group, if the following axioms are satisfied.

(i) Closure property $a * b \in G \ \forall \ a, b \in G$

(ii) Associativity :- The operation $*$ is associative on G i.e.,

$$a * (b * c) = (a * b) * c \quad \forall \ a, b, c \in G.$$

(iii) Existence of identity :- There exists an element $e \in G$, such that

$$a * e = a = e * a \quad \forall \ a \in G$$

e is called identity of $*$ in G.

(iv) Existence of inverse :- For each element $a \in G$, there exist an element $b \in G$, such that

$$a * b = e = b * a$$

The element b is called the inverse of element a with respect to $*$ and we write

$b = a^{-1}$

Abelian Group:- A group $(G, *)$ is said to be abelian or commutative if $a * b = b * a \ \forall\ a, b \in G$.

Note*: A group is not simply a set, but it is an algebraic structure.

* The smallest group for a given composition is the set $\{e\}$, containing identity elements.

* A group consisting of the identity element only, is called a trivial group; others are called non-trivial groups.

* If a group contains a finite number of elements, it is called a finite group.

* If the number of elements in a group is infinite, it is called an infinite group.

Order of a group:- The number of elements in a finite group is called the order of the group. ~~The~~ It is denoted by $O(G)$.

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An infinite group is called a group of infinite order.

e.g. (i) The set Z of integers is an infinite abelian group with respect to the operation of addition but Z is not a group with respect to the multiplication.

(ii) Let $G = \{1\}$, then G is an abelian group of order 1 with respect to multiplication.

(iii) Let $G = \{0\}$, then G is an abelian group of order 1 with respect to addition.

(iv) Let $G = \{1, -1\}$, then G is an abelian group of order 2 with respect to multiplication.